INFLUENCE OF GAS FLOW THROUGH THE VIBRATING GRID ON THE HYDRODYNAMICS OF A SHALLOW VIBROFLUIDIZED BED

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UDC 532.545

The dependence of the characteristics of vibrofluidization on the coefficients of hydraulic resistance of the bed and the grid and on a steady pressure drop (or gas flow) imposed from outside is analyzed.

It is known that in the process of vibrofluidization of granular beds an important role is played by the hydrodynamic interaction of the particles with the gas stream which penetrates the bed in the flight phase and retards its motion [1-5]. This interaction, whose influence is especially noticeable for finely dispersed beds, determines to a considerable extent the observed kinematic and dynamic parameters of a vibrofluidized bed (tossing height, effective expansion, phase angles of maximum tossing and falling of the bed on the grid, pressure pulsations in the bed, etc.).

Because of the very strong dependence of the hydrodynamic force on the porosity of the bed, in particular, its average value for the relatively dense bed which is rising relative to the grid is considerably greater than that for the falling bed which is able to expand. As a result, the time-averaged pressure drop in the bed proves to be different from zero; in the case when the vibrating grid is permeable, this leads to the appearance of a pumping effect in the bed [6-8]. Analogous phenomena result from the decrease in the hydraulic resistance of a granular bed to ascending flow and the increase in the resistance to descending gas flow upon the application of sufficiently intense vibrations (see [2, 5], as well as [9, 10]).

Other conditions being equal, the magnitude of the force of hydrodynamic interaction is determined by the effective velocity of gas filtration in the bed, which in turn depends on the pressure drop in it, i.e., on the pressure at the level of the grid when the pressure in the space above the bed is fixed. The pressure at the level of the grid obviously depends not only on the dynamic properties of the bed itself and the parameters of the vibrations but also on the conditions of penetration of gas into the bed from below, from the space below the grid. Hence, the physical nature of the very strong influence which the hydraulic resistance of the grid and the pressure created below it are able to exert on the characteristics of the vibrofluidization becomes understandable.

Such an influence is studied below on the basis of the physical model proposed in [11] for a horizontal granular bed on a grid vibrating in the vertical direction with an amplitude A and a frequency ω; the laboratory coordinate of the grid is $x_0 = A \sin \omega t$. The bed is assumed to be shallow, in the sense that the influence of wall friction and of the processes of propagation of stress waves in it is small, while the mode of vibrofluidization is mild, so that the bed is able to settle entirely onto the grid by the moment of the next separation and the collision of layers of granular material in flight does not occur. For simplicity we assume, as in [11], that the bed expands uniformly during the flight phase. This approximation allows us to avoid the laborious analysis of the propagation of porosity waves through the bed and is not very important (either in a qualitative or a quantitative respect) in a study of the motion of the bed and the pulsating pressure drop in it, but it becomes very rough in an analysis of phenomena whose very origin is connected with the process of expansion of the bed. Thus, the equations obtained below for the average pressure drop and gas flow due to the pumping effect must be considered as order-of-magnitude equations; this does not affect the qualitative aspect of the matter, however. We assume that the relative expansion of the bed is small. As follows from [11], for this the inequality $A/h_0 \ll 1$, which imposes a lower limit of $2h_0$ on the bed height, must be satisfied. The motion of the bed is studied in a one-dimensional statement.

Institute for Problems of Mechanics, Academy of Sciences of the USSR, Moscow. "Énergotsvetmet" Scientific-Industrial Society, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 4, pp. 604-613, April, 1978. Original article submitted May 11, 1977.

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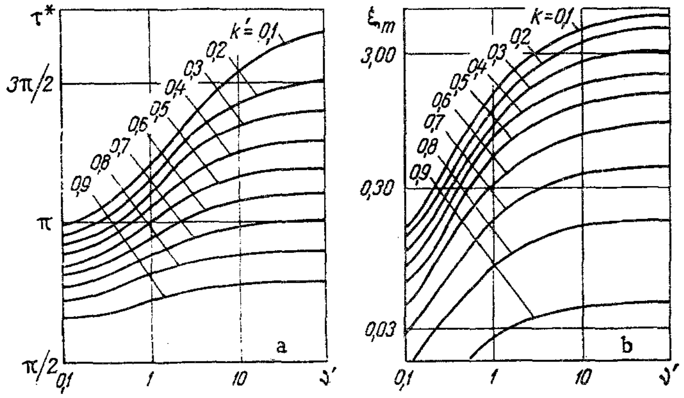

Fig. 1. Dependence of τ_* (a) and ξ_m (b) on ν' for different k' (numbers on curves).

To simplify the calculations, we assume that the hydraulic resistances of the bed and the grid are linear with respect to the gas filtration velocity. Then, taking the pressure in the space above the bed as the zero pressure reading and assuming that the pressure p_0 below the grid (which can be either positive or negative) is fixed, we write

$$p = K_1 Q,\ p_0 - p = K_2 q,\ K_1 = 2h_0\rho_0 K d_1 \beta,\ K = K(\rho_0), \tag{1}$$

where Q is the gas flow relative to the center of gravity of the bed while q is the gas flow through the grid (a plus sign corresponds to flows directed upward). The function $K(\rho)$ describes the influence of the constraint of the flow over the particles on the hydrodynamic force experienced by them while $\beta K(p)Q$ represents the force per unit mass of particles of the bed (see [11]). At a low Reynolds number a quasi-Stokes mode of flow is realized for an individual particle, so that $\beta = 9\mu/2a^2$; in the opposite case it is most convenient to treat β and $K(\rho)$ as empirical quantities. In this connection, we note that in the case under consideration the velocity of flow over a particle has the order ωA, and with intense vibrations it can be large, so that the Reynolds number will not be small even for fine particles. In the steady state (i.e., without vibrations) the gas filters through the bed with a filtration velocity q_0, where

$$p_0 = (K_1 + K_2) q_0. \tag{2}$$

We assume that this flow does not lead to fluidization of the bed in the usual sense; i.e., we impose the condition $q_0 < g/\beta K$.

Here, as in [11], we use the vertical coordinate z connected with the vibrating grid. Then, from the obvious relation $Q = -\dot{z}_b + q$ and from Eqs. (1) and (2), we obtain (here and below the dot denotes time differentiation)

$$q = -\varkappa Q + (1+\varkappa) q_0,\ Q = q_0 - \frac{\dot{z}_b}{1+\varkappa},\quad \varkappa = \frac{K_1}{K_2}. \tag{3}$$

Using (3), we obtain a problem analogous to that of [11] for the determination of the height of tossing of the center of gravity of the bed:

$$\ddot{\xi} + \dot{\xi}/\nu' = -k' + \sin\tau,\ \xi = \dot{\xi} = 0\ (\tau = \tau_1 = \arcsin k'). \tag{4}$$

Here we introduce the following dimensionless quantities:

$$\xi = \frac{z_b - h_0}{A},\ \tau = \omega t,\ \nu' = (1+\varkappa)\nu,\ \nu = \frac{\omega}{\beta K},$$
$$k' = \frac{g - \beta K q_0}{\omega^2 A} = k - k_q,\ k = \frac{g}{\omega^2 A},\ k_q = \frac{q_0}{\nu\omega A}. \tag{5}$$

As is easy to see, the formulation presented is correct if we neglect the effect of the small expansion of the bed on the trajectory of its center of gravity.

Thus, the permeability of the grid and the presence of flow imposed from outside lead to the same mathematical problem as that first analyzed by Kroll [1]; however, the modified parameters k' and ν' figure in it instead of the parameters k and ν usually used. The solution of the problem (4) is well known and can be represented in the form

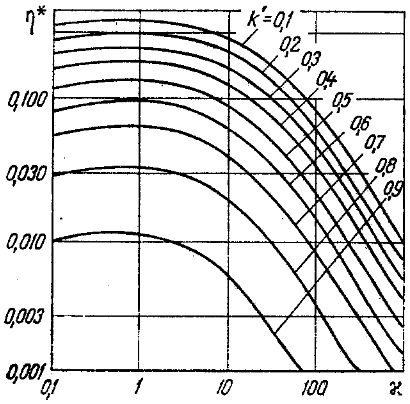

Fig. 2. Dependence of the quantity η^* at the moment of contact of the lower boundary of the bed with the grid on $\varkappa$ with $\nu = 0.1$ and different k'.

$$\xi(\tau) = \nu'(k'\nu' + \sqrt{1-k'^2})\left[1 - \frac{\nu'^2}{1+\nu'^2}\exp\left(-\frac{\tau-\tau_1}{\nu'}\right)\right] - $$

$$- k'\nu'(\tau-\tau_1) - \frac{\nu'^2}{1+\nu'^2}\sin\tau - \frac{\nu'}{1+\nu'^2}\cos\tau. \tag{6}$$

The condition of the onset of vibroboiling (separation of the bed from the grid) is also expressed in the usual form:

$$g - \beta K q_0 < \omega^2 A. \tag{7}$$

The maximum tossing height is reached at $\tau = \tau_*$, where τ_* is understood to be the next root of the equation $\dot{\xi}(\tau) = 0$ after τ_1. In Fig. 1 τ_* and $\xi_m = \xi(\tau_*)$ are presented as functions of ν' for different k'. It is seen that an increase in $\varkappa$ (in particular, a decrease in the hydraulic resistance of the grid at a fixed bed height or an increase in the height at a fixed K_2), as well as an increase in k_q (i.e., actually an increase in the pressure below the grid), involves the rising of the bed above the grid and strongly increases the tossing height, which is in agreement with the data of numerous experiments. As $\varkappa \to \infty$ the quantities τ_* and ξ_m approach finite limits, which depend on k'.

For the quantity $\eta = (h - h_0)/A$, which characterizes the expansion of the bed during its flight, we have a problem entirely analogous to that of [11]:

$$\ddot{\eta} + \frac{\sigma}{\nu}\dot{\eta} = \frac{1-\sigma}{(1+\varkappa)\nu}\left|\dot{\xi}\right|, \quad \eta = \dot{\eta} = 0 \ (\tau = \tau_1). \tag{8}$$

This problem was analyzed in [11] in the particular case of $\varkappa = 0$. The coefficient σ represents the ratio of the hydrodynamic force acting on a particle at the boundary of the bed facing the oncoming gas flow to the force acting on a particle in the interior of the bed at the same filtration velocity. The solution of (8) is easily written in analytical form using Eq. (6), in the same way as was done in [11]. It is not presented here in view of its cumbersome nature. In concrete calculations it is evidently more convenient not to use the analytical equation for η but to integrate the problem (8), or even both problems (4) and (8) at once, numerically with values of the parameters from (5) which are of interest.

The solution of (8) is greatly simplified in the limiting case of $\nu \ll 1$, when for η we obtain the simple relations

$$\eta = \frac{1-\sigma}{\sigma(1+\varkappa)}\xi, \quad \tau_1 \leqslant \tau \leqslant \tau_*, \tag{9}$$

$$\eta = \frac{1-\sigma}{\sigma(1+\varkappa)}(2\xi_m - \xi), \quad \tau_* \leqslant \tau \leqslant \tau'. \tag{10}$$

Here τ' is the next root after τ_1 of the equation $\xi(\tau) - \eta(\tau) = 0$ determining the phase angle of the contact of the lower boundary of the bed with the grid. When $\tau > \tau'$ the lower part of

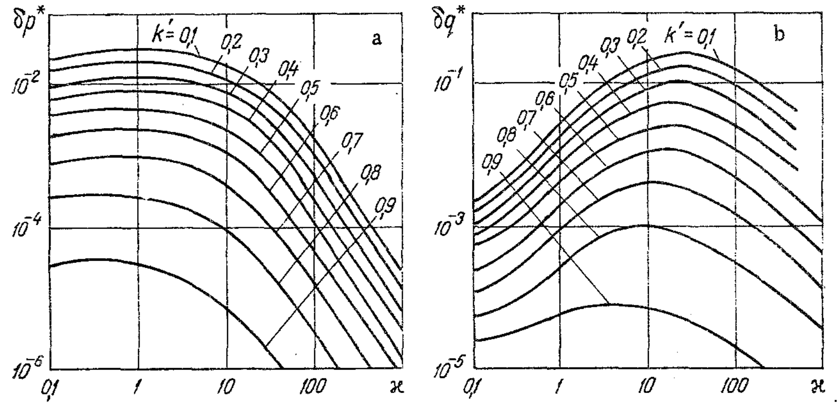

Fig. 3. Dependences of δp^* (a) and δq^* (b) on $\varkappa$ with $\nu = 0.1$ and different k'.

the bed is in a densely packed state, with the boundary separating this part from the particles in a suspended state moving rapidly upward with an increase in τ, reaching the upper surface of the bed at $\tau = \tau''$ determined from the equation $\xi(\tau) + \eta(\tau) = 0$. Exactly half of the bed is in the densely packed state at $\tau = \tau_2$, where τ_2 is determined from the equation $\xi(\tau) = 0$. When the expansions of the bed are not too great $\tau'' - \tau' \ll \tau_2 - \tau_1$. The condition for the realization of a mild mode of vibrofluidization has the form $\tau'' < \tau_1 + 2\pi$ and imposes restrictions on the region of variation of the parameters corresponding to this mode. The boundary of the packed region, as well as the dependences of the phase angles τ', τ_2, and τ'' on the parameters of the process, are easy to obtain, as in [11], on the basis of a study of the solutions of the problems for ξ and η presented above.

In order to analyze the influence of the parameters $\varkappa$ and k_q on the vibrofluidization process it is sufficient here to study only the limiting case of $\nu \ll 1$, when Eqs. (9) and (10) are valid. As is easy to see, the latter inequality can be satisfied for beds of sufficiently fine particles at not very high vibration frequencies. The results obtained with $\nu \ll 1$ are also valid in a qualitative respect in situations when this inequality is not satisfied. Since σ is a semiempirical parameter in need of experimental refinement, it is convenient to consider the quantity

$$\eta^*(\tau) = \sigma(1-\sigma)^{-1}\eta(\tau), \tag{11}$$

which does not depend on σ at all. The dependence of this quantity at $\tau = \tau'$ on the parameter $\varkappa$ with $\nu = 0.1$ and different k' is illustrated in Fig. 2. It is seen that in the region of large $\varkappa$ the expansion of the bed weakens monotonically with an increase in $\varkappa$; this conclusion already follows from the form of the curves for ξ_m in Fig. 1 and from Eq. (8). Similarly, it is easy to study the dependence of all the other quantities characterizing the vibrofluidization process (such as the pressure pulsations at the grid level) on $\varkappa$.

The equations presented above were obtained with the expansion of the bed neglected, when we simply took $\varepsilon = \varepsilon_0$ in calculating the hydrodynamic force. Actually, during the flight phase we have

$$1-\varepsilon = \rho \approx \rho_0(1 - A\eta/h_0), \tag{12}$$

from which it is easy to find, for example, the time average of the porosity of the bed

$$\langle \varepsilon \rangle = \varepsilon_0 + \frac{A\rho_0}{2\pi h_0}\int_{\tau_1}^{\tau_2} \eta\, d\tau \tag{13}$$

(here and below we allow for the presumed smallness of $\tau'' - \tau'$ in comparison with the flight duration $\tau_2 - \tau_1$).

With expansion of the bed neglected, the pressure at its lower boundary is $p_0 - K_2 q_0 = K_1 q_0$, as is easy to see from (1) and (2). This pressure can be calculated with allowance for expansion as follows. The relative velocity of gas filtration at the level of the bed which had the coordinate z_0 in the densely packed state is $Q - (z_0 - z_b)\dot{h}/h_0$, where the quantity Q is defined in (3). Therefore, the pressure gradient at this level is represented in the form (the uniformity of the expansion of the bed assumed here is taken into account)

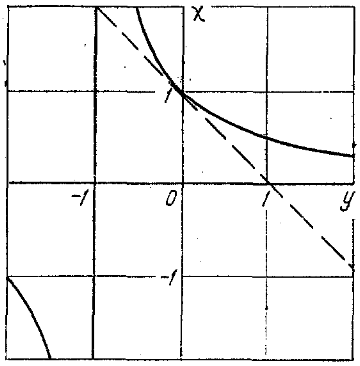

Fig. 4. Coefficients of variation of hydraulic resistance of a bed upon the application of vibrations: solid curve) $\varkappa_p$, $y = \delta q/q_0$; dashed curve) $\varkappa_q$, $y = \delta_q/\langle q\rangle$.

$$\frac{dp}{dz} = -\rho K(\rho) d_1 \beta \left(Q - \frac{z_0 - z_b}{h_0} \dot{h} \right). \tag{14}$$

The pressure at any level is easy to obtain by integrating (14) over dz from the upper boundary of the bed, where the pressure is equal to zero by convention, to the coordinate of the level using the equations presented above. It is seen, in particular, that the quantity (14) is nonuniform over the bed; i.e., with appreciable expansion of the bed the pressure profile in it differs from a linear profile.

By integrating (14) over the entire height of the bed, allowing for its uniformity (so that the term proportional to $\dot{h}$ vanishes entirely in the integration), and using the definitions of ξ and η and Eqs. (2), (3), and (12) and the condition $\rho h = \rho_0 h_0$ = const of conservation of granular material, with the accuracy of terms of the first order with respect to the small relative expansion of the bed we obtained the following expression for the pressure at the lower boundary of the bed:

$$p = 2h_0 \rho_0 K(\rho) d_1 \beta Q = K_1 \left(1 - \frac{A}{h_0} N\eta \right) \left(q_0 - \frac{A\dot{\xi}}{1+\varkappa} \right), \; N = \frac{\rho_0}{K} \left. \frac{dK(\rho)}{d\rho} \right|_{\rho=\rho_0}. \tag{15}$$

Averaging this quantity over a vibration period, we obtain the expression for the average pressure at the lower boundary:

$$\langle p \rangle = K_1 q_0 \left(1 - \frac{\rho_0 - \langle \rho \rangle}{\rho_0} \right) - \delta p, \tag{16}$$

where we used Eq. (13) and introduced the time-averaged pressure drop, corresponding to the situation with $q_0 = 0$ and calculated earlier in [11] in the particular case of $\varkappa = 0$:

$$\delta p = \left(\frac{\rho_0^2}{\pi} \frac{dK(\rho)}{d\rho} \right)_{\rho=\rho_0} d_1 \beta A^2 \delta p' = \frac{\rho_0 N}{\pi k \nu} d_1 g \delta p',$$

$$\delta p' = -\frac{1}{1+\varkappa} \int_{\tau_1}^{\tau_2} \eta \dot{\xi} d\tau. \tag{17}$$

Here we used the definitions of the parameters in (5) and (15). In the simplest case, when $\nu \ll 1$, after a simple calculation we obtain from (9) and (10)

$$\delta p' = \frac{1-\sigma}{\sigma} \delta p^*, \; \delta p^* = \frac{\xi_m^2}{(1+\varkappa)^2}, \tag{18}$$

from which it is seen that in this case the quantity δp is always positive; i.e., it corresponds to an average rarefaction below the bed. The dependence of the reduced pressure drop δp^* on $\varkappa$ with $\nu = 0.1$ and different k' is shown in Fig. 3a.

On the basis of (16) and (17) one can represent the time-averaged gas flow through the grid—bed system in the form

$$\langle q \rangle = \frac{p_0 - \langle p \rangle}{K_2} = q_0 + \delta q^\circ + \delta q,$$
$$\delta q^\circ = \varkappa \frac{\rho_0 - \langle \rho \rangle}{\rho_0} N q_0, \quad \delta q = \frac{\rho_0 N}{\pi k \nu} \frac{d_1 g}{K_2} \delta p'. \tag{19}$$

The term δq° describes the increase in flow due to the pressure drop imposed from outside owing to the expansion of the bed in flight and the corresponding decrease in its average hydraulic resistance, while the term δq characterizes the intensity of the pumping effect in the bed. With $\nu \ll 1$ we have approximately

$$\delta q^\circ = \frac{(1-\sigma) N}{2\pi\sigma} \frac{A q_0}{h_0} \frac{\varkappa}{1+\varkappa} \left[\int_{\tau_1}^{\tau_*} \xi d\tau + \int_{\tau_*}^{\tau_2} (2\xi_m - \xi) d\tau \right],$$
$$\delta q = \frac{\rho_0 N}{\pi k \nu} \frac{d_1 g}{K_1} \delta q', \quad \delta q' = \frac{1-\sigma}{\sigma} \delta q^*, \quad \delta q^* = \frac{\varkappa \xi_m^2}{(1+\varkappa)^2}. \tag{20}$$

The dependence of δq^* on $\varkappa$ with $\nu = 0.1$ and different k' is shown in Fig. 3b.

In the case when external flow q_0 is entirely absent the curves in Fig. 3 describe the average pressure drop and suction effect in an ordinary vibrofluidized bed. It is physically obvious that a decrease in the hydraulic resistance of the grid leads, on the one hand, to facilitation of the penetration of gas into the bed from the space below the grid, i.e., a decrease in δp, and, on the other hand, to intensification of the tossing of the bed above the grid and to an increase in its expansion, i.e., to an increase in δp. The competition of these opposing effects leads to the appearance of maxima in the curves of Fig. 3a. The additional gas flow δq due to the vibrations of the bed also reaches a maximum at a certain value of the parameter $\varkappa$, which evidently depends on k and ν. This flow is reduced to zero, i.e., the pumping effect vanishes, not only with an impermeable grid ($\varkappa = 0$), which is obvious, but also for systems in which the hydraulic resistance of the bed is far higher than the resistance of the grid ($\varkappa \to \infty$). This fact finds its natural explanation within the framework of the theory developed without the enlistment of artificial considerations of the type discussed in [12]. We note also that the results presented permit a detailed study of the dependence of δp and δq on the various dimensional parameters. For example, it is easy to see that in the case when the actual value of $\varkappa$ lies to the left of the maxima in the curves of Fig. 3a, an increase in the bed height, leading to an increase in $\varkappa$, causes an increase in the average pressure drop. At the same time, with higher h_0, such that the corresponding value of $\varkappa$ lies to the right of these maxima, an increase in the height of the bed leads to a decrease in the average pressure drop in it. Moreover, the latter at once makes it possible to explain the well-known disagreements in the opinions of various authors on the influence of the height of a vibrofluidized bed on the average pressure drop occurring in it. The dependence of the intensity of the pumping effect on the bed height has a similar nature.

In the case when there is an external gas flow through a granular bed, i.e., the pressure below the grid is different from zero, the application of vibrations leads to a change in the effective hydraulic resistance of the system under consideration. The coefficient of resistance is easily determined from (2) and (19), to wit,

$$K_e^{(p)} = \frac{p_0}{\langle q \rangle} = \frac{K_1 + K_2}{1 + \delta q^\circ / q_0 + \delta q / q_0}. \tag{21}$$

Thus, upon the application of vibrations the resistance to an ascending gas flow ($q_0 > 0$) decreases while the resistance to a descending flow increases when $\delta q > |\delta q^\circ|$. As $\delta q^\circ + \delta q \to -q_0$ the quantity (21) approaches infinity; this corresponds to the fact that flow through the bed ceases entirely, even though the pressure drop is different from zero. In this case a change in the sign of $K_e^{(p)}$ corresponds to a change in the direction of the gas flow.

A situation when the pressure below the grid is assigned as independent of the applied vibrations was considered above for determinacy. In the general case one must assign some connection between the values of the pressure and the gas flow, the character of which is determined by the construction of the apparatus used. The limiting situation opposite to that considered corresponds to the assignment of the gas flow through the system; in the adopted notation this corresponds to the quantity $\langle q \rangle$ being fixed.

In this case q_0 represents a quantity formally determined from Eq. (19), with δq being found in a way similar to what was done above. Then Eq. (2) for p_0 is valid as before. The corresponding coefficient of effective hydraulic resistance of the system has the form

$$K_e^{(q)} = \frac{p_0}{\langle q \rangle} = \frac{(K_1 + K_2)(1 - \delta q / \langle q \rangle)}{1 + \delta q^\circ / q_0}, \tag{22}$$

which differs considerably from (21). However, the resistance of the system decreases or increases as a function of the direction of the gas flow imposed from outside, and under certain conditions it can become negative in accordance with the experimental data in [2, 9, 10].

Depending on the concrete values of the parameters, limiting situations are possible when one of the flows δq° or δq is far greater than the other. If $\delta q^\circ \gg \delta q$ then the coefficient of variation

$$\chi_p = \frac{K_e^{(p)}}{K_1 + K_2} = \chi_q = \frac{K_e^{(q)}}{K_1 + K_2} = \frac{1}{1 + \delta q^\circ / q_0} \tag{23}$$

of the hydraulic resistance of the system due to the application of vibrations does not depend on the conditions under which the external gas flow is created. Conversely, in the case of $\delta q \gg \delta q^\circ$ we have

$$\chi_p = \frac{1}{1 + \delta q / q_0}, \quad \chi_q = 1 - \frac{\delta q}{\langle q \rangle}. \tag{24}$$

Consequently, the character of the variation in the resistance of the system upon the application of vibrations depends strongly on the experimental conditions, i.e., on the type of apparatus used, which must be taken into account in the treatment and interpretation of the data obtained. A comparison of the coefficients of variation of the resistance (24) is shown in Fig. 4. In principle there is no difficulty in numerical calculations of the dependence of these coefficients on the various physical and operating parameters.

NOTATION

A, amplitude of vibrations; a, particle radius; d_1, particle density; g, acceleration of gravity; h, half-height of bed; $K(\rho)$, function introduced into (1); K_1 and K_2, coefficients of hydraulic resistance of bed and grid, respectively; $K_e^{(p)}$ and $K_e^{(q)}$, coefficients of effective resistance of the system, defined in (19) and (20); k, inverse multiplicity of acceleration of vibrations; k_q, parameter in (5); N, parameter in (12); p and p_0, pressure above and below grid; δp, time-averaged pressure drop in bed; Q, gas filtration velocity relative to center of gravity of bed; q, gas flow through grid; δq, gas flow due to pumping effect; t, time; z, vertical coordinate, associated with the grid; β, reduced coefficient of resistance in (1); ε, porosity; η, relative expansion of bed; $\varkappa$, ratio of hydraulic resistances of bed and grid; μ, viscosity of gas; ν, dimensionless frequency of vibrations; ξ, dimensionless coordinate of center of gravity of the bed; ρ, bulk concentration of particles; σ, coefficient of boundary decrease in resistance; τ, dimensionless time (phase angle); ω, angular frequency of vibrations; the subscript 0 pertains to the densely packed state of the bed and to parameters of the gas flow in the ansence of vibrations; the upper prime and asterisk denote reduced quantities of different types; the angle brackets mean averaging over the vibration period.

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THE SCALE EFFECT IN THE LAMINAR FLOW OF DILUTE SOLUTIONS OF POLYMERS IN TUBES

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UDC 532.517.2:532.135

This paper describes the variation of the viscosity of polyoxyethylene solutions as a function of tube diameter. We investigate the effect of molecular weight and concentration of the polymer on viscosity anomalies.

1. Dilute solutions of some high-molecular-weight polymers have properties which make their hydrodynamic behavior different from that of ordinary liquids. The most striking difference is found in turbulent flow, when the hydrodynamic friction of the polymer solutions is only a fraction of the friction of the solvent. This effect was discovered by Toms in connection with the flow of solutions of polymethylmethacrylate in monochlorobenzene [1]. The phenomenon of reduced turbulent friction resistance attracted the attention of many researchers, and a large number of studies on this subject have already been published.

Subsequently Toms established an anomaly in the laminar flow of polymer solutions [2]. It was found that in the laminar flow of solutions in tubes the value of the viscosity depends on the tube diameter. Toms regarded this as proof of the existence of effective slippage along the wall, to which Oldroyd [3] attributed the reduced-resistance effect. The scale anomalies discovered in this connection did not attract any special attention on the part of investigators. The only study worth mentioning is [4], the authors of which, in particular, noted difficulties in explaining the results on the basis of slippage along the wall.

In the mid-1960s it was found that high-molecular-weight polymers such as polyhydroxyethylene and polyacrylamide were capable of reducing turbulent friction at solution concentrations of a few parts per million. Unlike the liquid used in Toms' experiments, the viscosity of solutions at such concentrations differs by only a few percent from the viscosity of the solvent. Apparently this is why no more interest was shown in the anomalies of laminar viscosimetric flows. In investigations of the reduced resistance, researchers confined their attention to measurements of the viscosity of solutions using ordinary viscosimeters with thin capillaries, assuming that dilute polymer solutions in laminar flow behave like Newtonian liquids. Even when differences were again observed in the values of the viscosity at different diameters of the measuring segments of the viscosimeters, no importance was attached to this fact, on the assumption of possible degradation of the solutions in thin tubes or of errors in measurement [5].

This disregard of the anomalies observed in viscosity measurements did not introduce any substantial distortion into the data on turbulent resistance but led to erroneous conclusions concerning the effect of polymer additives on the stability of circular Couette flow. In a number of published reports it was asserted that polymer additives increase the value of the critical Taylor number [6-10]. As was explained in [11], this conclusion was due to the fact that in determining the Taylor number, researchers used solution viscosity values measured by means of thin capillary viscosimeters. It was shown that if the viscosity values used are those obtained by means of viscosimeters with fairly large diameters in the measuring segments, the critical values of the Taylor number for dilute polymer solutions coincide with those of Newtonian liquids. Thus, it became clear that the viscosimetric anomalies should no longer be ignored, since that might lead to further errors.

Institute of Problems in Mechanics, Academy of Sciences of the USSR, Moscow. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 34, No. 4, pp. 614-621, April, 1978. Original article submitted April 19, 1977.

0022-0841/78/3404-0409$07.50